Effect of multiphoton absorption and free carriers in slow-light photonic crystal waveguides

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We examine the effects of multiphoton absorption, free carriers, and disorder-induced linear scattering in slow-light photonic crystal waveguides. We derive an analytic formulation for self-phase modulation including the group velocity scaling of the nonlinear phase shift in materials limited by three-photon absorption as a representative nonlinear process. We investigate the role of free carriers and derive an approximate critical intensity at which these effects begin to strongly modify the optical field. This critical intensity is employed to determine an optimal group index for the self-phase modulation in the slow-light devices. These observations are confirmed with numerical modeling.

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Slow-light enhanced nonlinearities have been studied extensively in semiconductor photonic crystals [1,2] as key elements to future photonic technologies. A variety of nonlinear processes, such as temporal soliton compression [3], four-wave mixing [4], Raman scattering [5,6], third-harmonic generation [7], and self-phase modulation (SPM) [8,9] have been demonstrated in photonic crystal waveguides (PhCWG). A key challenge in these works is multiphoton absorption mechanisms, such as two-photon absorption (TPA) and three-photon absorption (ThPA), which restrict the desirable Kerr effect. Additionally, TPA and ThPA generate free carriers that induce both free-carrier absorption (FCA) and free-carrier dispersion (FCD). This latter effect is particularly detrimental to the propagating pulse shape. Here we present an analytic formulation of slow-light SPM for materials limited only by ThPA and compare it to TPA-restricted materials. We derive critical intensity thresholds, I_c, at which FCD degrades the pulse propagation. Though the analysis here focuses on the $1.55\,\mu$m wavelength range, the results in this work are applicable to any material waveguide system limited by ThPA. In particular, several groups have recently initiated systematic investigation of nonlinear optics in silicon near $2\,\mu$m where TPA is drastically reduced [10,11].

The propagation of picosecond optical pulses in a waveguide with suppressed TPA and negligible group velocity dispersion is governed by [9,12,13]

$$\frac{\partial E}{\partial z} = -\frac{\alpha}{2}E + ik_0 n_2 |E|^2 E - \frac{\alpha_3}{2}|E|^4 E + \left(ik_o \frac{dn}{dN} - \frac{\sigma}{2}\right) N_c E, \tag{1}$$

where E is the electric field envelope, α the linear loss, $k_0 = 2\pi/\lambda$, n_2 the optical Kerr coefficient, α_3 the ThPA coefficient, dn/dN the index change per carrier density, σ the FCA coefficient, N_c the number of carriers, and z the distance along the waveguide of length, L. In the low intensity and approximately picosecond pulse limit, we evaluate the case of negligible carriers, $N_c = 0$. Substituting $E = \sqrt{I(z)}\exp[i\phi(z)]$ into Eq. (1), the solutions for intensity I and phase ϕ are

$$I_{\text{out}}(L, I_0, t) = I_0 \frac{e^{-\alpha L}}{[1 + 2\alpha_3 I_0^2 L_{3\text{eff}}]^{1/2}}, \tag{2}$$

$$\phi(L, I_0, t) = \frac{k_o n_2}{\sqrt{\alpha\alpha_3}} \arctan[\gamma],$$
$$\gamma = \frac{2L_{3\text{eff}} I_o \sqrt{\alpha\alpha_3}}{1 + \exp(-\alpha L)[1 + 2\alpha_3 I_o^2 L_{3\text{eff}}]^{1/2}}, \tag{3}$$

where $I_0 = P_0/A_{3\text{eff}}$, P_0 is the peak power in the waveguide, $A_{3\text{eff}}$ is the effective area for third-order processes such as Kerr, $L_{3\text{eff}} = (1 - \exp[-2\alpha L])/(2\alpha)$. We take $A_{3\text{eff}} \approx 0.2\,\mu\text{m}^2$, though it is larger at larger n_g [9,14].

The phase solution follows the general form of SPM: $\phi(I_0) = k_0 n_2 I_0 L_{\text{eff}}$. In conventional optical fibers with small nonlinearities, $L_{\text{eff}}(\text{lin.}) = (1 - \exp[-\alpha L])/\alpha$, e.g., no intensity dependence. In the case of materials with nonlinear absorption, different definitions of effective length should be used: $L_{\text{eff}}(\text{TPA})$ [12] or ThPA, $L_{\text{eff}}(\text{ThPA})$, readily obtainable from rearranging Eq. (3), which we will use below. We now extend the formalism to include slow group velocity. Though slow-light effects physically affect the field intensity, here we attach the scalings to the coefficients for notational simplicity: [Kerr] $n_{2\text{eff}} = n_2(n_g/n_0)^2$; [TPA] $\alpha_{2\text{eff}} = \alpha_2(n_g/n_0)^2$ [2]; [ThPA] $\alpha_{3\text{eff}} = \alpha_3(n_g/n_0)^3(1/A_{5\text{eff}})^2$ [9], with $A_{5\text{eff}}$ the fifth-order area and n_g the group index. We assume material dispersion of the nonlinear susceptibilities are negligible within the wavelength range as compared to the slow-light scalings: $n_2(\text{Si, GainP}) = 6 \times 10^{-18}\,\text{W/m}^2$, $\alpha_2(\text{Si}) = 1 \times 10^{-11}\,\text{m/W}$, $\alpha_3(\text{GaInP}) = 3 \times 10^{-26}\,\text{m}^3/\text{W}^2$. The linear scattering loss scales as $\alpha_{\text{eff}} = \alpha(n_g/6)^2$, as is the case with disorder-induced coherent loss [15], an ultimate physical limit of PhCWGs. We note that a linear scaling is appropriate away from the Brillouin zone edge and at lower group indices [14]. In the end, the

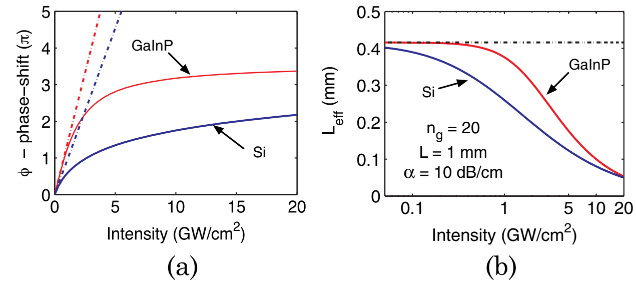

Fig. 1. (Color online) (a) Phase shift, ϕ, as a function of I_o for (i) GaInP and (ii) Si. The straight dashed lines are α only, while the curves shows ϕ impacted by TPA or ThPA. (b) Effective lengths, $L_{\rm eff}(\mathrm{TPA})[\mathrm{Si}]$ and $L_{\rm eff}(\mathrm{ThPA})[\mathrm{GaInP}]$ versus I_o with $L_{\rm eff}(\mathrm{lin.})$ shown (dashed black line) as reference.

details of the scaling do not impact the results qualitatively, and we focus on the context of nonlinearities. The factor of 6 is chosen based on our samples, which often have this value as a minimum n_g.

In Fig. 1(a), we plot phase shift, ϕ, as a function of I_0 for representative materials, GaInP (ThPA) [3,9], and silicon (TPA) [8], with $n_g = 20$, $L = 1\,\mathrm{mm}$, and $\alpha = 10\,\mathrm{dB/cm}$, achievable in PhCWGs at present. The solid curves indicate the analytic formulation of ϕ according to the pertinent formula, while the straight (dashed) lines serve as a reference in the absence of nonlinear loss. The ThPA-limited material GaInP demonstrates larger phase shift compared to the TPA-limited Si. In Fig. 1(b), we show the corresponding $L_{\rm eff}$ versus I_o. While $L_{\rm eff}(\mathrm{TPA})$ deviates almost immediately, note that $L_{\rm eff}(\mathrm{ThPA}) \approx L_{\rm eff}(\mathrm{lin.})$ up to about $I_0 = 0.3\,\mathrm{GW/cm^2}$ for $n_g = 20$, and is still 90% of $L_{\rm eff}(\mathrm{lin.})$ at $1\,\mathrm{GW/cm^2}$, indicating that nonlinear losses are weak under these conditions. This is in sharp contrast to the TPA material, which falls off immediately. While multiphoton absorption restricts SPM, and other nonlinear effects in general, the far greater impediment is free-carrier effects, which we now examine.

Indeed the assumption of low intensity used to derive Fig. 1 breaks down as the phase reaches a plateau and one must consider carrier effects. The two dominant phase-shift mechanisms in the optical pulses are Kerr ($ik_0 n_2 I_0$) and FCD ($ik_0 \frac{dn}{dN_c} N_c$) as illustrated in Eq. (1), with redshifts and blueshifts, respectively. To achieve "small" pulse distortion from FCD, we set the criteria $\frac{dn}{dN} N_c \approx n_2 I_0$, that is, Kerr effects are roughly balanced by FCD. The carrier equation, $\frac{\partial N_c(z,t)}{\partial t} = \frac{\alpha_3}{3\hbar\omega}|E(z,t)|^6 - \frac{N_c(z,t)}{\tau_c}$, describes free carriers generated by ThPA, as well as recombination with lifetime, τ_c. In waveguide geometries in silicon, various authors have reported τ_c on the order of hundreds of picoseconds [8,16]. If τ_c is much greater than the pulse duration T_0, and low repetition rates do not allow accumulation of carriers from successive pulses, the last term is negligible and we estimate the number of carriers generated at the waveguide input by integrating the carrier equation over the pulse duration, T_0, assuming a Gaussian pulse shape: $N_c(t) = \frac{\alpha_3 I_0^3 T_0}{3\hbar\omega}$. The critical intensity at which ThPA-induced FCD begins to play a significant role in the pulse dynamics is

$$I_c = \sqrt{\frac{n_2 3\hbar\omega}{\alpha_3 T_0 \frac{dn}{dN}} \frac{1}{\left(\frac{n_g}{n_0}\right)}}. \qquad (4)$$

Note that for TPA, with N_c from [12], the corresponding value is $I_c = \frac{3}{4}\frac{n_2 2\hbar\omega}{\alpha_2 T_0 \frac{dn}{dN}} \frac{1}{\left(\frac{n_g}{n_0}\right)}$. The effect of repetition rate on pulse dynamics with free carriers was investigated and well described in [12]. We plot I_c as a function of n_g for $T_0 = 2\,\mathrm{ps}$ in Fig. 2(a). The intensity levels employed in recent experimental demonstrations of slow-light SPM in silicon [8] and GaInP [9] are also included. Indeed in [8], the spectra are blueshifted as expected from $I_o > I_c$. The ThPA-limited material (GaInP) is predicted to achieve 40% greater intensity before carrier effects occur as compared to the TPA material (Si). We now plot in Fig. 2(b) the minimum length to achieve a π phase shift in SPM in GaInP at I_c, defined as $L_{\rm crit,min}$. The minimum critical length $L_{\rm crit,min}$ decreases with larger n_g and smaller α. The spike in $L_{\rm crit,min}$ is a cutoff above which a π phase shift cannot be achieved due to excessive loss. The dip, on the other hand, suggests that there is an optimal n_g for a given set of sample parameters due to the competing slow-light modifications of nonlinearities, carrier effects, and the disorder scattering losses. This plot elucidates the need to reduce linear scattering, in addition to suppressing nonlinear loss, to capture the full benefits of slow-light. In particular, if losses could be reduced to $\alpha_{\rm eff} = 1\,\mathrm{dB/cm}$ in the "fast-light" region ($n_g = 6$), group indices in excess of $n_g = 100$ could readily be achieved, opening up tremendous opportunities for slow-light devices. Recent developments in "loss engineering" open a potential way forward [14].

To examine the carrier effects further, we numerically solve the nonlinear Schrödinger equation in the two materials for identical conditions, $I_o = 2\,\mathrm{GW/cm^2}$ and $n_g = 20$, corresponding to $2I_c$ in GaInP and $2.8I_c$ in Si.

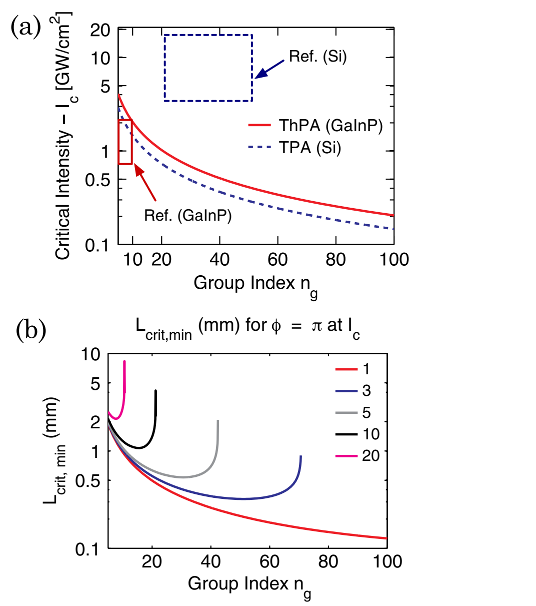

Fig. 2. (Color online) (a) Critical intensity as a function of n_g. Boxed regions indicated recent slow-light SPM measurement ranges [8,9]. (b) Estimated minimum critical length $L_{\rm crit,min}$ to achieve $\phi = \pi$ as a function of n_g.

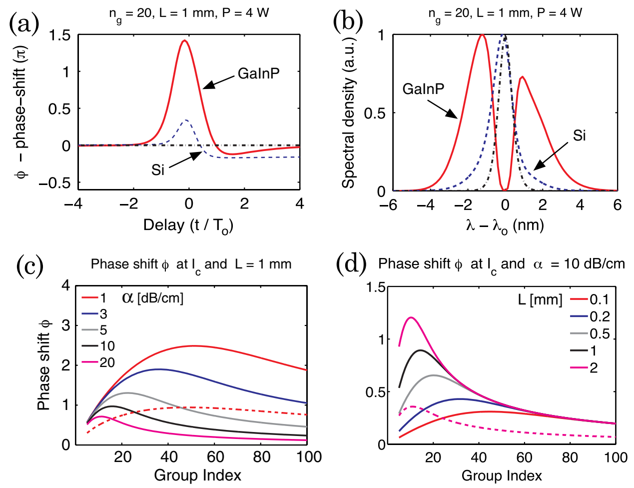

Fig. 3. (Color online) (a) Phase shift (π) versus delay for $P = 4\,\mathrm{W}$ at $n_g = 20$ for GaInP, solid, and Si, dashed. (b) Spectra corresponding to panel (a). (c) Phase shift ϕ versus n_g for a fixed length ($L = 1\,\mathrm{mm}$) and variable scattering loss, α_{eff} in GaInP. The critical intensity I_c (Fig. 2), γ_{eff}, and α_{eff} scale with group index. The peak in the various curves demonstrate an optimal n_g shifting toward smaller n_g for increasing α_{eff}. (d) Same as (c), except with $\alpha_{\mathrm{eff}} = 10\,\mathrm{dB/cm}$ and variable L.

We plot the temporal pulse properties in Fig. 3(a). While $\phi = 1.5\pi$ is observed with only a slight onset of blueshift for GaInP, the pulse undergoes a dramatic blueshift in Si. The spectral properties in Fig. 3(b) show slight asymmetry in GaInP, as expected above I_c; however, the center wavelength remains in place. A small phase shift with a trailing blue component are apparent in the case of Si.

We now further examine the point of optimal n_g at which the desired nonlinear effects are enhanced, while the linear and nonlinear losses are relatively weaker. In order to elucidate this we plot ϕ at I_c as a function of n_g for GaInP in Fig. 3(c) for several values of α_{eff}. Presently many groups are able to achieve propagation losses around $\alpha_{\mathrm{eff}} = 10\,\mathrm{dB/cm}$ around $n_g = 6$, with $n_g(\mathrm{opt.}) \sim 20$ for a $L = 1\,\mathrm{mm}$ device. At increased α_{eff}, the optimal group index shifts toward smaller values as the scattering losses begin to dominate at lower group indices. Importantly, this trend demonstrates that even though slow-light enhanced nonlinear losses play a role at larger group indices, the role of linear scattering is perhaps more important, even at modest group indices. The red-dashed curve represents silicon for the case of optimal loss. Strikingly, the TPA device with $\alpha_{\mathrm{eff}} = 1\,\mathrm{dB/cm}$ is equivalent to a ThPA device with $\alpha_{\mathrm{eff}} = 10\,\mathrm{dB/cm}$ in terms of maximum phase shift.

We also consider the effect of varying L by employing the same parameters, with the exception of α_{eff} fixed at $10\,\mathrm{dB/cm}$, in Fig. 3(d). The dashed line once again indicates the optimal TPA case with $L = 2\,\mathrm{mm}$ equivalent to about a $0.2\,\mathrm{mm}$ ThPA device. The behavior of increased L and increased α_{eff} is similar, as expected from the $\alpha_{\mathrm{eff}}L$ loss dependence. The motivation for short slow-light devices is thus twofold: (i) small footprint for on-chip devices for photonic integrated circuits, and (ii) avoiding slow-light scattering losses. It is apparent that devices limited by TPA must be much longer than ThPA devices to achieve a given nonlinear effect. One cannot simply increase power, as free-carrier effects begin to dominate. We note this phenomenon is a general feature of slow-light enhanced nonlinear effects [5–7].

We investigated fundamental limits and design space of nonlinear effects in slow-light waveguides due to multiphoton absorption, free-carrier effects, and linear scattering. We applied these metrics to the nonlinear SPM process in materials limited by ThPA and determined that there is an optimal n_g in slow-light devices. Our analysis suggests that enhanced linear scattering in slow-light structures is an equally important loss mechanism as compared with nonlinear absorption. While the results in this work are illustrated in the context of SPM in GaInP and Si, they can readily be extended to other material platforms and nonlinear processes.

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