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# Frequency microcomb stabilization via dualmicrowave control

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Optical frequency comb technology has been the cornerstone for scientific breakthroughs in precision metrology. In particular, the unique phase-coherent link between microwave and optical frequencies solves the long-standing puzzle of precision optical frequency synthesis. While the current bulk mode-locked laser frequency comb has had great success in extending the scientific frontier, its use in real-world applications beyond the laboratory setting remains an unsolved challenge due to the relatively large size, weight and power consumption. Recently microresonator-based frequency combs have emerged as a candidate solution with chip-scale implementation and scalability. The wider-system precision control and stabilization approaches for frequency microcombs, however, requires external nonlinear processes and multiple peripherals which constrain their application space. Here we demonstrate an internal phase-stabilized frequency microcomb that does not require nonlinear second-third harmonic generation nor optical external frequency references. We demonstrate that the optical frequency can be stabilized by control of two internally accessible parameters: an intrinsic comb offset  $\xi$  and the comb spacing  $f_{\rm rep}$ . Both parameters are phase-locked to microwave references, with phase noise residuals of 55 and 20 mrad respectively, and the resulting comb-to-comb optical frequency uncertainty is 80 mHz or less. Out-of-loop measurements confirm good coherence and stability across the comb, with measured optical frequency instability of  $2 \times 10^{-11}$  at 20-second gate time. Our measurements are supported by analytical theory including the cavity-induced modulation instability. We further describe an application of our technique in the generation of low noise microwaves and demonstrate noise suppression of the repetition rate below the microwave stabilization limit achieved.

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hase-stabilized optical frequency combs (OFCs), with the multitude of coherent and stable spectral lines, bridges the research frontiers in ultrastable laser physics and ultrafast optical science.<sup>1-6</sup> Phase stabilization requires two-dimensional feedback control on the comb's intrinsic two degrees of freedom. the comb spacing and one of the comb line optical frequencies. While the comb spacing can be readily measured with a highspeed photodetector, assessment of the comb line optical frequency fluctuations often requires non-trivial and/or nonlinear processes. One approach is to compare the OFC against an external optical reference, and previous phase stabilization of Kerr frequency comb has been predominantly demonstrated with schemes based on this approach.<sup>7,8</sup> The requirement of an external optical reference, however, limits the achievable compactness of Kerr frequency comb and impairs its integration of chip-based photonics with electronics. Another approach is to devise a nonlinear optical interferometry which reveals the optical frequency instability through the so-called carrier-envelope-offset frequency  $f_{ceo}$ , an internal OFC property resulting from the difference in the phase and group velocities.<sup>9</sup> Knowledge of  $f_{rep}$  and  $f_{ceo}$  fully determines the optical frequencies of a mode-locked laser-based OFC, and phase locking them to stable microwave references ensures the intricate stability of the optical frequency synthesizer. Figure 1a shows the schematic of a state-of-the-art f-2f nonlinear interferometer widely adopted to measure the  $f_{ceo}$ .<sup>10</sup> First, the output pulse from a mode-locked laser is spectrally broadened in a highly nonlinear photonic crystal fiber such that its optical spectrum spans more than an octave. Then the octave-level spectrum is separated into two parts: the lowerfrequency end undergoes second-harmonic generation in a nonlinear crystal while the higher-frequency end only experiences free-propagation. Finally, the two beams are put together in both transverse and longitudinal coordinates for them to interfere on a photodetector and generate a beat note at  $f_{ceo}$ . For the nonlinear processes to work properly, spectral broadening, in particular, few-cycle pulses with peak powers in the 10-kW level are required.6

While the microresonator-based OFC, or Kerr frequency microcomb, is approaching the performance of mode-locked laser-based OFC in many aspects,<sup>11-35</sup> its output pulse duration and peak power are still lower by orders of magnitude. Application of f-2f and 2f-3f nonlinear interferometer technique to the Kerr frequency comb is thus challenging and power demanding. The pulse duration can potentially be improved by finer dispersion engineering, but the peak power is fundamentally limited by the bandwidth-efficiency product<sup>36</sup> and the large comb spacing. On the other hand, the 10 to 100 GHz comb spacing of Kerr frequency comb is considered an advantageous feature for applications like coherent Raman spectroscopy,37 optical arbitrary waveform generation,<sup>38</sup> high bandwidth telecommunication,<sup>39,40</sup> and astrospectrograph calibration.<sup>41-43</sup> In a recent pioneering demonstration of self-referenced Kerr frequency comb where f-2fnonlinear interferometer technique is adopted,27,44 a hybrid approach was utilized with two interlocking combs, a THz spacing comb with dispersive waves at f and 2f is used to calculate  $f_{ceo}$ while a second relatively closely spaced comb is simultaneously generated to measure  $f_{rep}$ . This approach is successful; however, the experimental setup includes several components including lasers at different wavelengths, frequency shifters, and a thulium amplifier. In addition, in recent years there have been several experiments attempting to facilitate stabilization and reduce the size, weight, and power (SWaP) impact of peripherals via more compact control,45 electro-optic modulation of large spacing combs,<sup>46</sup> and micromachined atomic cells.<sup>47,48</sup>

In this work, we attempt to extend current stabilization techniques using the unique generation mechanism of Kerr frequency microcombs. We demonstrate comb stabilization using only internal degrees of freedom in a single resonator with no external nonlinear processes and achieve an Allan deviation (AD) of  $2 \times 10^{-11}$  at a 20-s gate time for the stabilized comb lines. After the stabilization of  $f_{rep}$ , we show that  $\xi$  resembles  $f_{ceo}$  in gauging the optical frequency instability without the need of an external optical reference.  $\xi$  is specifically sensitive to the fluctuation in pump frequency, which is at the same time the 0<sup>th</sup> comb line frequency. Phase locking of  $f_{rep}$  and  $\xi$  to low-noise microwave oscillators thus guarantees the optical frequency stability of the microcomb. This method has potential for chip-scale integration, while circumventing the need for a large number of peripherals, thereby preserving the key SWaP advantage of frequency microcombs.

#### **Results and discussion**

Kerr frequency comb formation in the microresonator is illustrated in Fig. 1b. The intracavity power is gradually increased by decreasing the frequency of an initially blue-detuned pump. As coupled cavity power crosses a threshold, modulation instability (MI) gain dominates over cavity loss forming primary comb lines via degenerate four-wave mixing (FWM). The frequency difference between the primary lines  $(\Delta)$  is determined by dispersion, pump power and mode interaction. In general, however,  $\Delta$  need not be an integer multiple of the cavity repetition rate  $f_{rep}$ . Thus, on the formation of subcombs with secondary comb lines spaced by  $f_{rep}$ , around the primary lines, the generated comb exhibits an intrinsic offset frequency  $\xi$ , that may be directly detected by a photodetector.<sup>13-16</sup> While in general the comb state can be complex with multiple offset beats or chaotic (see Supplementary Note I), with detuning and pump power control it is possible to generate just a single set of primary lines (unique  $\Delta$ ) and therefore a microcomb with a well-defined  $\xi$  uniquely. Figure 1c shows the optical spectrum of such a comb in the C-band, the particular phase-locked breather state generated necessitates a modulated comb spectrum since we do not have full merging of the subcombs. Figure 1d zooms in to a 3 nm bandwidth showing the spacing  $\Delta$  and the merging of subcombs. In Fig. 1e, we see the resultant RF beat notes at  $\xi = 523.35$  MHz and  $f_{rep} = 17.9$  GHz, as detected by a high-speed photodetector. We further confirm the existence of only one primary comb family and the uniformity of  $f_{\rm rep}$  and  $\xi$  across the Kerr frequency comb by measuring the beat notes at different spectral segments with a tunable 0.22-nm bandpass filter, in Fig. 1f and g, respectively. The breather state generated is stable and exists across a range of powers and detuning. In our specific instance, we see a stable breather over a detuning span of over GHz and with a power tolerance of  $\approx 1$  dB. In addition to being of use in locking, such phase-locked breather Kerr combs have also recently come under some scrutiny for their rich cavity dynamics.49

### Experimental characterization of microcomb full stabilization.

Figure 2a depicts the frequency microcomb setup for stabilization.<sup>50</sup> Detailed descriptions of the chip fabrication and measurement setup are included in the "Method" and Supplementary Notes II and III, respectively. The Si<sub>3</sub>N<sub>4</sub> microresonator is fabricated with CMOS-compatible processes and the spiral design ensures that the relatively large resonator fits into a tight field-of-view to avoid additional cavity losses introduced by photomask stitching and discretization errors. The resonator has a quality factor Q of 1.2 million intrinsically in the transverse-electric mode polarization, with near-critical coupling for a 600,000 loaded Q. The waveguide width of 2 µm (725 nm height) allows for significant mode overlap between the fundamental and first-order TE modes, and thereby the resonator exhibits periodic



mode-interaction spaced by 4 nm. Free-space to chip coupling is implemented by a  $600 \,\mu\text{m}$  long adiabatic coupler which allows, with our coupling free-space lens, a total chip coupling loss not more than 5.5 dB. In order to suppress environmental temperature fluctuations from the microcomb, the resonator chip is placed on a thermoelectric cooler for thermal control and placed in a box with two layers of thermal foam insulation. We note that

the box is not entirely sealed, which gives little convective currents within the box or between the external environment and the box, leading to some temperature fluctuations. The entire setup including the optics is then placed in an acrylic chamber. Acoustic noise is dampened by placing the enclosed setup on a sorbothane sheet and then placing it on an active optical table. The comb spacing of 17.9 GHz is directly measurable by sending **Fig. 1 Phase-locked breather comb generation and characterization. a** Schematic of the current f-2f nonlinear interferometry to measure  $f_{ceo}$ . By comparing the higher-frequency comb segment and the second-harmonic of the lower-frequency segment, a beat note at  $f_{ceo}$  with sufficient signal-to-noise-ratio (SNR) is generated on a photodetector (PD). This often requires comb spectral broadening in a highly nonlinear fiber and/or a broad octave-level comb span. **b** The unique generation mechanism of frequency microcombs provides an alternative full-stabilization route that does not require external nonlinear processes. Modulation instability and four-wave mixing then generates the initial comb lines with  $\Delta$  spacing, and subsequently secondary lines with  $f_{rep}$  spacing. Often,  $\Delta$  is not an integer multiple of  $f_{rep}$ . The frequency microcomb therefore has an offset frequency  $\xi$  innately. As elaborated later,  $\xi$  resembles  $f_{ceo}$  in directly gauging the optical frequency instability. **c** Example frequency microcomb spectrum showing subcombs around primary lines yet to fully merge. **d** Primary comb lines are formed with a spacing ( $\Delta$ ) of 1.1 nm and then generate overlapping subcombs. **e** Electrical spectrum measures two distinct beat notes of  $f_{rep} = 17.9$  GHz and  $\xi = 523.35$  MHz, confirming the presence of a unique  $\xi$  (**f**) and (**g**) Free-running change in  $f_{rep}$  and  $\xi$  at different spectral regions are measured to be the same within error bars of  $\approx 2$  kHz and  $\approx 200$  kHz, respectively. At wavelengths where the beat notes have SNR higher than 10 dB (100 kHz RBW), 10 measurements are taken to determine the mean values of the comb spacing and offset frequencies. Measurement error bar is the peak-to-peak deviation from 10 measurements.

the output to a high-speed photodetector. The comb spacing is then phase locked and stabilized to a microwave oscillator by controlling the pump power through a fiber electro-optic modulator (primary loop) and either the gain of the erbium-doped fiber amplifier (slow loop marked in yellow) or temperature of the chip mount (slow loop marked in green). Quality of the  $f_{\rm rep}$ stabilization is detailed in Supplementary Note III.

Of note, the free-running offset frequency  $\xi$  is much noisier than the comb spacing  $f_{rep}$  due to the additional multiplier in the constitutive equation that is proportional to the spacing between the primary comb lines and pump ( $\Delta$ ) divided by the repetition rate (the brackets in Eq. (1) correspond to the floor operation):

$$\xi = \Delta - \left| \frac{\Delta}{f_{\rm rep}} \right| f_{\rm rep} \tag{1}$$

To this end,  $f_{rep}$  stabilization loop is always engaged before measurements on the offset frequency is conducted. Comparison between free-running and post- $f_{rep}$  stabilization  $\xi$  is included in Supplementary Note III. As the offset frequency is localized to the spectral region where secondary comb lines overlap, a 0.22 nm optical bandpass filter is used to select the overlapped comb lines around 1553.5 nm for detection. The beat note is thus improved to 50 dB above the noise floor with a resolution bandwidth (RBW) of 10 kHz, sufficient for a reliable feedback stabilization (more than 45 dB with 10 kHz RBW). The offset frequency is divided by 15 before it is phase locked and stabilized to a microwave synthesizer. The pre-scaling reduces the phase fluctuation, while preserving the instability of the oscillator frequency, and thus it makes the  $\xi$  phase-locked loop more robust against noise. The high-bandwidth feedback on  $\xi$  is achieved by direct current modulation of the external cavity diode laser (ECDL), and the slow feedback is done through piezoelectric transducer control of the ECDL. We heterodyne beat our Kerr frequency comb with a stabilized fiber frequency comb (FFC) to measure out-of-loop stability. All microwave oscillators and frequency counters are commonly referenced to a rubidiumdisciplined crystal oscillator with a  $5 \times 10^{-12}$  frequency instability at 1 s integration time.

Kerr frequency comb generation mechanism can be described by the nonlinear Schrödinger equation and the cavity boundary condition:  $^{\rm 51-53}$ 

$$\frac{\partial E^{n}(z,t)}{\partial z} = -\frac{\alpha}{2}E^{n}(z,t) - \mathrm{i}\frac{\beta_{2}}{2}\frac{\partial^{2}E^{n}(z,t)}{\partial t^{2}} + \mathrm{i}\gamma|E^{n}(z,t)|^{2}E^{n}(z,t),$$
(2)

$$E^{n+1}(0,t) = \sqrt{1-T}E^n(L,t)\exp(\mathrm{i}\varphi_0) + \sqrt{T}E_\mathrm{i}, \qquad (3)$$

where  $E^n(z,t)$  is the electric field envelope function at the *n*th cavity round-trip, *z* is the propagation, *t* is the retarded time,  $\alpha$  is cavity round-trip loss,  $\beta_2$  is the group velocity dispersion (GVD),

 $\gamma$  is the nonlinear coefficient, T is transmission coefficient of the coupler, and  $\varphi_0$  is the phase accumulated in a round-trip. Here the microresonator is assumed to be critically coupled, for simplicity. Under the mean-field approximation and the good cavity limit, the primary comb spacing, which depends on the optimal frequency where modulation instability gain reaches its maximum, can be solved as (Supplementary Note IV):

$$\Delta = \frac{1}{\sqrt{\pi c |\beta_2|}} \sqrt{\eta \left( n_{\rm g} f_{\rm p} - N \frac{n_{\rm g}^2}{n_0} f_{\rm rep} - \frac{\gamma c P_{\rm int}}{\pi} \right)} \tag{4}$$

where  $\eta = \frac{\beta_2}{|\beta_2|}$  is the sign of the GVD,  $n_g$  is the group index,  $n_o$  is the refractive index, N is the longitudinal mode number, c is the speed of light in vacuum,  $f_p$  is the pump frequency, and  $P_{int}$  is the intracavity pump power. This picture of comb formation is illustrated in the schematic Fig. 2b.

Equations (1) and (4) explicitly show the dependence of  $\xi$  on  $f_{\rm p}$ ,  $f_{\rm rep}$ , and  $P_{\rm int}$ . In the high-Q Si<sub>3</sub>N<sub>4</sub> microresonator,  $P_{\rm int}$  is resonantly enhanced to be as high as 30 W and it is the dominant heat source to change the cavity temperature and subsequently the comb spacing.<sup>29</sup> For instance, a pump power variation of 0.12% results in a microcomb line-to-line frequency spacing of  $1.6 \times 10^{-5}$  fluctuation, corresponding to a large cavity temperature fluctuation of 1 K. While  $f_{\rm rep}$  is directly dependent on cavity temperature and  $P_{\rm int}$ .<sup>54</sup> we note that  $f_{\rm rep}$  is only indirectly dependent on  $f_{\rm p}$ . This indirect dependence is eventually attributable to a change in  $P_{\rm int}$  since a change in detuning changes the power coupled to the cavity.

Theoretically this can be understood by noting that the usual way  $f_p$  directly contributes to changes in  $f_{rep}$  is via Raman selffrequency shift,<sup>54</sup> however since our comb is not a soliton, this effect is negligible. Thus, we expect the  $f_{rep}$  stabilization will effectively eliminate the P<sub>int</sub> fluctuation. Under this assumption, the offset frequency is reduced to just a function of pump frequency once the comb spacing is stabilized. Control of  $f_{rep}$  and  $\xi$  is thus equivalent to the regulation of  $f_{
m rep}$  and  $f_{
m ceo}$  in full stabilization of the Kerr frequency comb. Figure 2c plots the measured and simulated offset frequency as a function of pump wavelength after the  $f_{rep}$  stabilization (The simulated slope in Fig. 2c is described in detail in Supplementary Note IV). We observe that the offset frequency scales linearly with the pump wavelength at a slope of 4.5 MHz per picometer shift of pump (corresponding to a sensitivity of  $3.7 \times 10^{-2}$ ). In addition, in Fig. 2d, we introduce an out-of-loop perturbation to pump power after  $f_{rep}$  stabilization, but observe no change in the breather tone. If the  $f_{rep}$  lock had not entirely eliminated the introduced power change, the  $P_{int}$  dependence of  $\Delta$  (and hence  $\xi$ ) would have caused a change in the breather frequency. The measurements therefore validate the assumption that  $f_{rep}$  stabilization effectively



**Fig. 2 Dual-microwave control comb stabilization principle. a** Frequency microcomb setup for stabilization. Here EOM, Electro-optic modulator; EDFA, Erbium-doped fiber amplifier. For  $\xi$  and  $f_{rep}$  stabilization, an EOM controls the pump power and the ECDL diode current controls the pump laser frequency, through two phase-locked loops. We also note that, in place of the EOM, a polarization rotator with a PBS can also serve for intensity modulation. To increase the locking duration, we implemented slow control of the ECDL PZT (red dashed line) and the EDFA gain (yellow dashed path). The slow feedback to the EDFA gain may also alternatively be replaced by sending the feedback signal to the temperature controller on the chip holder (which controls device temperature) instead (green dashed path). This reduces loop dynamic range but also mitigates the effects of ambient temperature drift. The stability of the locked microcomb is interrogated out-of-loop by beating with an independently stabilized fiber frequency comb. **b** Frequency domain illustration of the demonstrated full-stabilization technique. Here the offset frequency,  $\xi$ , is linked with the primary comb line spacing,  $\Delta$ , by the constitutive relation

 $\xi = \Delta - \left| \frac{\Delta}{f_{rep}} \right| f_{rep}$ . Furthermore,  $\Delta = \frac{1}{\sqrt{\pi c |\beta_2|}} \sqrt{\eta \left( n_g f_p - \overline{N_{n_0}^{\frac{2}{p}} f_{rep} - \frac{\gamma c P_{int}}{\pi} \right)}}$  where  $\beta_2$  is the group velocity dispersion (GVD),  $\eta = \frac{\beta_2}{|\beta_2|}$  is the sign of the GVD,  $n_g$  is the group index,  $n_o$  is the refractive index, N is the longitudinal mode number, c is the speed of light in vacuum,  $\gamma$  is the nonlinear coefficient, and  $P_{int}$  is the intracavity pump power. **c** Subsequent to  $f_{rep}$  stabilization, the offset frequency shows strong linear correlation with pump wavelength (blue circles) with a slope of about 4.5 MHz per picometer shift of pump. The simulated slope (plotted in orange) also shows good agreement with measured data. **d** Measured offset frequency remains constant within the error bar, verifying that  $f_{rep}$  stabilization effectively eliminates the intracavity pump power fluctuation. For (**c**) and (**d**), 10 measurements are taken to determine the mean value, and the error bars are defined as the peak-to-peak deviations from the 10 measurements. Here the pump frequency is not yet stabilized, resulting in the error bars in the offset frequency measurements.

eliminates the intracavity pump power fluctuation and reduces the dependence of  $\xi$  to just a function of pump frequency. Mode hybridization in the current multi-mode Si<sub>3</sub>N<sub>4</sub> microresonator leads to abrupt increase of local GVD and results in the pinning of primary comb lines.<sup>55,56</sup> The effect reduces the slope, *i.e.* sensitivity, of offset frequency in gauging the pump frequency fluctuation (Eq. 4). Nevertheless, the sensitivity is already more than two orders of magnitude larger than the optical frequency division ratio,  $\partial f_{\rm rep}/\partial f_{\rm opt} \sim 10^{-4}$ , where  $f_{\rm opt}$  is the optical frequency of any the generated comb lines. Thus the fluctuations of the Kerr frequency comb lines  $\delta f_{\rm opt} = \frac{1}{\partial f_{\rm rep}/\partial f_{\rm opt}} \delta f_{\rm rep} + \frac{1}{\partial \xi/\partial f_{\rm opt}} \delta \xi$  ( $\delta \xi$  is under constant  $f_{\rm rep}$ ) are thus bounded by the residual error and the local oscillator of the  $f_{\rm rep}$  stabilization loop



**Fig. 3 Characterization of**  $\xi$  **feedback loop. a** Electrical spectrum of the stabilized beat note of  $\xi$  with a resolution bandwidth (RBW) of 10 Hz. To minimize the crosstalk between the two phase-locked loops, here the proportional-integral corner frequency is set lower than that of the  $f_{rep}$  loop. Furthermore, a second integrator at 500 Hz and a differentiator at 100 kHz are included to better suppress low-frequency noise and improve the loop stability, respectively. **b** Single-sideband (SSB) phase noise of the reference 523.35 MHz local oscillator and the residual loop error, showing excess phase noise of the stabilized  $\xi$  above 2 kHz from carrier. **c** To verify the uniformity of the offset frequencies,  $\xi$  are measured at two other spectral regions (marked in red; 1544.72 nm and 1547.86 nm) beside the 1553.5 nm region where the beat note is stabilized to 523350000 Hz in the phase-locked loop. The selected spectral segments are representative as each  $\xi$  is generated from the overlap of different groups of secondary comb lines. (**d**) and (**e**) Counter results and the corresponding histogram analysis (insets). The mean value at 1544.72 nm is 523349999.84 Hz, the standard deviation over 160 measurements is 390 mHz, and the interquartile range is 40 mHz.

(Supplementary Note III), when both  $f_{\rm rep}$  and  $\xi$  are stabilized. We must note here that although the coefficient of the  $\delta\xi$  term is relatively small, if  $\xi$  is not locked at all then the pump is still free to drift (in this situation  $d\xi$  will be orders of magnitude larger than  $df_{\rm rep}$ ) and  $f_{\rm opt}$  is no longer stable.

**Characterization of the proposed stabilization technique**. Figure 3a, b shows the quality of the  $\xi$  stabilization (after  $f_{\rm rep}$  stabilization is engaged). To minimize the crosstalk between the two phase-locked loops, here the proportional-integral corner frequency is set lower than that of the  $f_{\rm rep}$  stabilization loop. On the other hand, a second integrator at 500 Hz is included to better suppress low-frequency noise. Compared to the unstabilized beat note, the stabilized  $\xi$  shows a clear resolution limited coherent spike (Fig. 3a). The noise oscillation at 205 kHz is the remaining

crosstalk derived from the corresponding noise peak in the  $f_{\rm rep}$  stabilization loop (Supplementary Note III). Figure 3b. plots the single-sideband phase noise of the reference oscillator as well as the residual error of our feedback loop from 1 Hz to 1 MHz. While the low-frequency noise is well suppressed to below the reference, excessive phase noise above 2 kHz from carrier is observed. The root mean square phase error integrated from 6 Hz to 600 kHz is 55 mrad. To verify the uniformity of the offset frequencies,  $\xi$  are measured at two distinct spectral regions other than 1553.5 nm where the beat note is used for stabilization. The selected spectral segments (marked red in Fig. 3c) are representative as each  $\xi$  is generated from the overlap of different groups of secondary comb lines. Counter results and the corresponding histogram analysis are summarized in Fig. 3d, e. The mean values at 1544.72 nm and 1547.86 nm are 523349999.84 Hz

and 523349999.92 Hz, respectively, while the beat note at 1553.5 nm is stabilized to 523350000 Hz. Offset frequencies at different spectral regions are identical within a sub-Hz error, confirming the uniformity of  $\xi$  across the Kerr frequency comb. Phase locking of  $f_{\rm rep}$  and  $\xi$  to low-noise microwave oscillators is complete and it should guarantee the optical frequency stability of the Kerr frequency comb.

#### Out-of-loop assessment of the stabilized Kerr frequency comb.

We interrogated the locked microcomb by beating with an external stabilized FFC, and counting the beat frequencies with a 10-digit,  $\Lambda$ -type frequency counter. The FFC is independently stabilized with the f-2f interferometer technique (Supplementary Note V). In Fig. 4a an external perturbation is artificially introduced by disconnecting the slow feedback to the laser piezo control and instead using the piezo to induce a periodic 20 MHz frequency fluctuation. The inset shows clear suppression of the external perturbation (>20 dB) when both phase locked loops are engaged. In Fig. 4b we plot the Allan deviations (ADs) of the comb lines under two different locking schemes. When slow feedback is provided to the EDFA and there is no ambient temperature stabilization (yellow path in Fig. 2a), a  $5 \times 10^{-11} / \sqrt{\tau}$ (at 1 s) frequency instability is observed, close to the 17.9 GHz reference oscillator. No apparent difference is observed between the ADs of the two comb lines 43 nm apart, indicating a good coherence transfer across the Kerr frequency comb. For longer gate times, the ADs show a characteristic linear dependence on the gate time that can be attributed to the uncompensated ambient temperature drift. For instance, considering the current chip holder has a long-term temperature stability of less than 10 mK which is limited by the resolution of the temperature sensor, a pump power proportional change of  $1.2 \times 10^{-5}$  is needed to keep the intracavity temperature and consequently the  $f_{rep}$ constant. Such pump power variation in turn results in a change of 13 kHz in the pump frequency  $(\Delta f_p = \frac{\gamma c}{\pi n_g} \Delta P_{int}$  from Eq. (4)). The frequency instability is gauged to be in the range of  $7 \times 10^{-11}$ when compared to optical carrier of ≈188 THz, in agreement with the asymptotic behavior of the measured AD. We can however partially compensate this ambient temperature drift via improved double-walled packaging along with slow feedback to a TEC on the chip holder that directly controls chip temperature (green path in Fig. 2a). After implementing these improvements we achieve an improved Allan deviation of  $2 \times 10^{-11}$  at 20-s gate time, and decreased the slope of AD increase from  $\tau$  to  $\tau^{0.23}$ , marked as the blue line in Fig. 4b.

The unique stabilization technique thus implemented can be used to stabilize the absolute frequency of each comb line in the Kerr frequency comb without the need of an octave-level comb spectrum and any external nonlinear process. We also confirm the universality of this method by finding comb states with unique  $\xi$  across multiple rings with widely varying f<sub>rep</sub> and waveguide geometries (detailed in Supplementary Note VI). However, this method does not allow us to determine the precise optical frequency of each line without calibration via an optical reference. Despite this apparent limitation, the method described here can be used to extend the functionality of frequency combs to various applications while preserving a low SWaP as external nonlinear processes are not required for comb stabilization. We briefly describe one such application and lay out a path for its achievement.

**Generation of low-noise microwaves**. As another application of the correlation between  $f_{p}$ , pump power,  $f_{rep}$  and  $\xi$ , we propose the generation of low-noise microwaves by stabilizing the pump frequency and  $\xi$ . The following method may also be applied to

full-stabilization of combs with  $f_{\rm rep}$  too large to directly measure. In prior literature low-noise microwave generation via optical frequency division<sup>57,58</sup> has been accomplished with broad octave-level combs that allow for internal detection of  $f_{\rm ceo}$  via f–2f interferometry and atomic transition or external cavity reference that may be used to stabilize a single comb line.<sup>59–63</sup> The comb  $f_{\rm rep}$  instability would then be suppressed by a factor close to the optical frequency division ratio ( $\approx 10^4$ ) when compared to the optical reference instability. We can however remove the requirement for the detection of  $f_{\rm ceo}$  and hence for the comb spectrum to be across an octave in frequency if we instead use a modified method based on the stabilization of  $\xi$  described previously.<sup>64</sup>

To generate low-noise microwaves, we propose locking the pump to a stable optical reference and then locking the frequency  $\xi$  to a microwave LO, it can be shown that doing so stabilizes the  $f_{\rm rep}$ . This is because  $\xi$  depends on both pump frequency and pump power and upon stabilizing the pump frequency to an optical reference, pump power is the sole factor determining the stability of both  $\xi$  and  $f_{rep}$  which are now directly correlated. Figure 4c shows the setup schematic for stabilization. The device is pumped with a high power EDFA and a comb state measured to have a unique offset  $\xi$  at 40 MHz frequency with over 50 dB SNR (at an RBW of 100 kHz) is generated. The pump is then locked to a fully-stabilized fiber frequency comb referenced to an ultrastable cavity. The offset  $\xi$  is locked via feedback to a polarization rotator. Together with the PBS placed after the EDFA, this can modulate the input power to the device. To adjust the chip mount and resonator temperature, we feedback with slow bandwidth to the TEC. This partially suppresses ambient thermal drift and increases lock dynamic range. We confirm the strong linear correlation between pump power,  $f_{\rm rep}$  and  $\xi$  in Fig. 4d. We observe that for a 3% change in pump power, corresponding to a 200 kHz change in  $f_{rep}$ , there is a 3 MHz change in  $\xi$ . This implies that the frequency sensitivity of  $\xi$  to pump power fluctuations ≈15 times higher, and (due to the large difference in the carrier frequencies) the frequency sensitivity of  $\xi$ to pump power is  $\approx 6700$  times higher than that of  $f_{rep}$ . Locking  $\xi$ may thus suppress phase noise in  $f_{rep}$ , beyond what would be achieved by directly locking  $f_{rep}$  to a microwave reference. When  $f_{\rm rep}$  is divided down (in the limit of no residual noise) to the carrier frequency of  $\xi$ , this would correspond to a phase noise suppression of 64.7 dB.  $f_{rep}$  before and after  $\xi$  lock is recorded on the ESA and plotted in Supplementary note VII, confirming that locking  $\xi$  suppresses  $f_{rep}$  noise. In Fig. 4e we plot the phase noise of both the locked  $\xi$  and  $f_{rep}$  (divided down to 40 MHz), we see that at high offset frequencies the noise is suppressed by over 60 dB as per our expectations, however there is still uncompensated  $f^2$  thermal noise due to the coupled ambient temperature fluctuations, that our loops cannot remove. This noise can however be further reduced by suppressing  $\xi$ 's phase noise parameters, and increasing the thermal isolation or implementing passive temperature stabilization techniques such as an auxiliary laser<sup>34</sup>

#### Conclusion

Utilizing only internally accessible comb-parameters such as  $f_{\rm rep}$  and offset frequency  $\xi$ , we have demonstrated an approach to fully stabilize comb line frequencies of appropriately generated comb states. The existence of microcombs with only one set of primary comb lines, critical for the new stabilization method, is a consistent property of these nonlinear microresonators and have been found in devices across multiple chipsets with very different dispersion, Q and  $f_{\rm rep}$ . The sensitivity for our device is measured as  $3.7 \times 10^{-2}$ , already more than two orders of magnitude larger



Fig. 4 Characterization of fully-locked comb stability and low-noise microwave generation. a Optical beat frequency between the pump and the stabilized fiber frequency comb (FFC). With both the  $f_{rep}$  and  $\xi$  phase-locked loops engaged, the artificially introduced pump frequency perturbation (red curve) is suppressed and the optical beat frequency remains constant (black curve). The inset plots the corresponding power spectral densities, showing a more than 20 dB pump frequency noise suppression by the stabilization loops. **b** Free-running comb Allan Deviation (AD) is plotted with black hexagons. The orange and brown circles plot the AD of the pump and ith comb line, respectively, when stabilized with slow feedback to the preamp (yellow path in Fig. 2a). No apparent difference is observed between the ADs of the two comb lines 43 nm apart, indicating a good coherence transfer across the microcomb. Our measured values are close to the local oscillator AD (gray pentagons) used to stabilize frep. For longer gate times, ADs show a characteristic linear dependence on the gate time, attributed to the uncompensated ambient temperature drift. To mitigate this, improved environmental isolation and partial compensation of ambient temperature drift (via green path in Fig. 2a) are implemented and AD of stabilized pump with is plotted in blue squares. The AD is improved to  $2 \times 10^{-11}$  at 20-s gate time, and slope of AD increase is reduced from  $\tau$  to  $\tau^{0.23}$ . The shaded region on the right marks the point where slow thermal drift degrades the lock performance. c The setup schematic to generate low-noise microwaves. The pump laser is directly locked to an optical reference, in this case, a fully stabilized FFC referenced to an ultrastable cavity. Subsequently the offset  $\xi$  is also locked via feedback to pump power (through the action of the polarization rotator and PBS). The above-mentioned loops indirectly lock f<sub>rep</sub> due to suppression of both pump frequency noise and power fluctuations. Slow feedback is sent to the temperature of the chip mount via a TEC to partially suppress ambient thermal drift. **d**  $f_{rep}$  is plotted in the black bars and  $\xi$  is plotted in blue. Both  $f_{rep}$  and  $\xi$  are strongly correlated with pump power, with measured correlation greater than 0.997. The absolute and frequency sensitivity to pump power are, respectively,  $\approx$ 15 and  $\approx$ 6,700 times larger for  $\xi$  than  $f_{rep}$ . **e** Locking of both  $\xi$  and pump frequency allows for suppression of  $f_{rep}$  noise (also see Supplementary Note VII). Here, the phase noise of the locked  $\xi$  (at 40 MHz) is plotted in black and the measured frep, after engaging both feedback loops (and carried down to 40 MHz), is plotted in blue. We observe a 62 dB suppression of noise at high offset frequencies matching well with our expectations, in the unshaded region to the right; however, at lightly shaded region in the center uncompensated 1/f<sup>2</sup> thermal noise (plotted in the dashed red line) begins to dominate and eventually surpasses the locked signal in the shaded region on the left. We can mitigate the effect of this thermal noise via better environmental isolation or passive cavity temperature feedback.

than the optical frequency division ratio, and it can be improved by novel microresonator designs to suppress the mode hybridization.<sup>63</sup> Furthermore, having both  $f_{rep}$  and  $\xi$  phase-locked to low-noise microwave oscillators concurrently enables the frequency microcomb optical stability. Simulation results of the correlation between  $\xi$  and pump frequency, needed for a successful lock, are also in good agreement with the experiment. We further show the frequency microcomb has frequency instability of  $2 \times 10^{-11}$  at 20-s gate time, bounded by the external microwave reference. For gate times longer than 20 s, AD increases due to the uncompensated ambient temperature drift. Such long-term drift can be improved by a better thermal shield or a more effective temperature control.<sup>65</sup> We have also discussed an avenue in which this method might prove useful, namely in the generation of low-noise microwaves, and we have also demonstrated a proof-of-principle experiment showing this. We believe our method could find use in a range of applications that require stable chip-scale OFCs due to its advantages of low SWaP and potentially reduced need for optical peripherals.

#### Methods

**Si<sub>3</sub>N<sub>4</sub> microresonator fabrication**. First a 3 µm thick oxide layer is deposited via plasma-enhanced chemical vapor deposition (PECVD) on p-type 8" silicon wafers to serve as the under-cladding oxide. Then low-pressure chemical vapor deposition (LPCVD) is used to deposit a 725 nm silicon nitride for the spiral resonators, with a gas mixture of SiH<sub>2</sub>Cl<sub>2</sub> and NH<sub>3</sub>. The resulting silicon nitride layer is patterned by optimized 248 nm deep-ultraviolet lithography and etched down to the buried oxide layer via optimized reactive ion dry etching. The sidewall is observed to have an etch verticality of 85°. Next the silicon nitride spiral resonators are over-cladded with a 3 µm thick oxide layer and annealed at 1200 °C. The refractive index of the silicon nitride film is measured with an ellipsometric spectroscopy from 500 nm to 1700 nm. The fitted Sellmeier equation assuming a single absorption resonance in the ultraviolet,  $n(\lambda) = \sqrt{1 + \frac{(2.9065 \pm 0.00192)\lambda^2}{(1.4505007 \pm 1.03964)^2}}$ , is imported into the COMSOL Multiphysics for the waveguide dispersion simulation, which includes both the material dispersion and the geometric dispersion.

Stabilization setup and out-of-loop analysis. The PI2D control servos we use for feedback in both  $f_{rep}$  and  $\xi$  phase-locked loops have a full bandwidth of 10 MHz and can be set to have two PI corners, to effectively suppress low-frequency noise, in addition to a PD corner to increase the loop stability. To ensure minimal crosstalk between the loops, the PI corners are set at very different frequencies. For the  $f_{rep}$  stabilization, the PI corner for the first integrator is set to 200 kHz while the second integrator is switched off. For the  $\xi$  stabilization, the PI corners are set to 500 Hz and 50 kHz to achieve higher suppression for low-frequency noise. In addition, the PD corners are set to 200 kHz and 100 kHz, respectively, with a differential gain of 10 dB. The derivative control is important in our system to make the feedback loop more stable and achieve optimal noise suppression. Due to alignment drift in the optics, the mean level of the servo output keeps increasing until the lock is lost in a few minutes. To increase the operation time, we also include in each loop a slow feedback where the feedback error signal is generated by integrating the servo output for 1 s. The control units of the slow feedback loops are the EDFA gain and the piezoelectric transducer (PZT), which have larger dynamic ranges than the EOM and the diode current. For out-of-loop analysis, the beat frequency between the Kerr frequency comb and the fiber laser frequency comb is counted with a 10-digit, A-type frequency counter and the

Allan deviation is estimated using the equation  $\sigma_A(\tau) = \sqrt{\frac{1}{N}\sum_{i=1}^{i=N} {(\bar{y}_{i+1}-\bar{y}_i)^2 \over 2}}$ , where  $\tau, \bar{y}_i$ , and  $N = \min\left\{20, \left[\frac{200}{\tau}\right]\right\}$  are the gate times, the fractional frequencies, and the number of samples, respectively. The grating-based filter critically removes the unwanted reference fiber laser frequency comb teeth such that clean heterodyne beat notes with more than 30 dB signal to noise ratio (measured with a 100 kHz RBW), sufficient for reliable counting measurements, can be routinely obtained.

#### Data availability

The datasets generated during and/or analyzed during the study are available from the corresponding authors on reasonable request.

#### Code availability

The code used in the analysis of the datasets is available from the corresponding authors on reasonable request.

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#### Author contributions

S.W.H. designed the experiment and A.K. conducted the experiment. S.W.H. and J.Y. designed the microresonator. M.Y. and D.-L.K. performed the device nanofabrication. S. W.H. and A.K. analyzed the data. S.W.H., A.K., and C.W.W. contributed to writing and revision of the manuscript.

#### **Competing interests**

The authors declare no competing interests.

#### Additional information

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# **Supplementary Information**

# Frequency microcomb stabilization via dual-microwave control

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## **Supplementary Note I: Other comb states**

The comb state is generated with a New Focus Velocity TLB-6730 as the pump laser. The pump frequency is tuned with a 2 MHz step via fine piezo control. The general multiple mode-spaced (MMS) scheme of comb formation involves the generation of several subcomb families with incommensurate spacing between them [SR1, SR2]. This is illustrated in Supplementary Fig. S1a, as we might expect, combs evolving via this scheme would, in general, produce several low frequency RF beats. The comb state we stabilize however, is one with a single offset beat and has just one other subcomb family aside from the sub-comb around the primary comb line as illustrated in Supplementary Fig. S1b. This state is not a necessary part of the comb evolution process and is only observed under the right conditions of power and detuning. Here we briefly describe several other states that we observe in our microresonator. One of the comb states we have observed, generates an equally spaced set of beats spanning around 600 MHz. This 'RF comb' is shown in Supplementary Fig. S2a, in this particular case an interesting point to note is that although multiple subcomb families exist in this state, the RF beats being equally spaced indicates a relationship between the different subcomb families.

As detuning is changed this state changes to one with higher noise that does not show a regular equally spaced comb structure in the RF domain, as shown in Supplementary Fig. S2b. This state then eventually evolves into one with continuous low frequency noise, the RF spectrum at the repetition rate of such a comb is shown in Supplementary Fig. S2c. In addition, we observe states

similar to the one we use for stabilization, having a strong low frequency RF beat in addition to the beat due to  $f_{rep}$ , as shown in Supplementary Fig. S2d, but exhibiting slightly different behavior with regards to degree of correlation between pump and the offset beat.



Supplementary Figure S1. a, The general MMS scheme of comb formation, the two sets of subcombs, shown in blue and green belong to different families because the two sets of primary comb lines around which the subcombs form, are generated independently by the pump. The first set of primary comb lines are formed at an offset of  $\Delta_1$  from the pump and the second set are formed at an offset of  $\Delta_2$  from the pump, since  $\Delta_2$  is not a multiple of  $\Delta_1$  and neither  $\Delta_2$  nor  $\Delta_1$  need be integral multiples of the  $f_{rep}$ , there are two offset beats generated by beating of subcombs with each other, these offset beats are shown in the schematic as  $\xi_1$  and  $\xi_2$ . Now if this idea is extended to multiple subcomb families we would expect the generation of multiple RF beatnotes, (and if the subcombs were broad enough we would also generate harmonics of the beatnotes) and this is what we experimentally observe. b, A special case of MMS comb formation that results in the generation of a single RF beat note (aside from the beat due to  $f_{rep}$ ) that corresponds to the offset  $\xi$ between subcombs. Note that, in this case, only the first set of primary comb lines is formed due to modulation instability via the pump, all other primary comb lines are generated via cascaded four-wave mixing between the pump and the first set of primary comb lines, this mechanism allows for a single offset  $\xi$ , throughout the comb. We choose to stabilize this particular state due to the strong correlation between the pump frequency and  $\xi$  due to the dependence of  $\xi$  on  $\Delta$ , as described in the main text.



**Supplementary Figure S2. a,** Multiple RF beats spanning 600 MHz with a spacing of 16 MHz generated by a comb state. The beats being equally spaced indicates that there is correlation between the offsets of different subcomb families. **b**, Multiple RF beats spanning over a GHz, generated by a comb state. Lack of defined structure to the beats suggests a general MMS scheme for the evolution of the state **c**, RF spectrum showing continuous low frequency noise, this state is obtained from the state in b. by changing the detuning such that the number of RF beats keeps increasing till we eventually have a 'noise pedestal' of continuous noise. **d**, RF spectrum showing a strong offset beat (breather tone) along with multiple harmonics. This state is similar to the one we stabilize; except for the fact that it is less stable to change in pump power or detuning (there is a sudden transition to another state). It also exhibits different behavior with regards to degree of correlation between pump frequency and offset beat.



Supplementary Note II: Microresonator dispersion and comb spectrum

**Supplementary Figure S3.** Waveguide dispersion is calculated taking into account of both the material dispersion and the geometric dispersion. **a**, Refractive index  $n_0$ , measured at 1.81 at pump wavelength of 1598 nm. **b**, Group index  $n_g$ , measured at 2.064 at the pump wavelength. **c**, Group velocity dispersion (GVD) measured at 23 fs<sup>2</sup>/mm at the pump wavelength. **d**, Third-order dispersion (TOD) measured at 265 fs<sup>3</sup>/mm at pump wavelength of 1598 nm.



**Supplementary Figure S4.** Example filtered C-band frequency microcomb spectrum. Formation of primary comb lines with  $\Delta = 1.1$  nm and overlap between secondary comb lines are observed (left inset). Its electrical spectrum measures two distinct beat notes of  $f_{rep} = 17.9$  GHz and  $\xi = 523.35$  MHz (right inset). The highly modulated spectrum is due to mode disruptions every 4 nm, which periodically perturbs the GVD.

## Supplementary Note III: Details of the measurement setup

The measurement setup is shown in Fig. 2 in the main text. The comb spacing is measured by sending a section of the comb to a high speed photodetector to directly detect the beat note from

the repetition rate  $f_{rep}$ . We then obtain the error signal for feedback by downmixing the output signal with a 17.9 GHz local oscillator. This error signal is the input to a PI<sup>2</sup>D lock box with a bandwidth of 10 MHz, which sends the feedback signal to an EOM to modulate the input power of the 3W EDFA which pumps the microresonator. The EDFA is operated in the current control mode to achieve effective modulation of the output power, to within 1% and less than 0.1 dB. Even with the free-space alignment optics, the lock can be maintained for more than an hour in each measurement set. In a fully packaged system, the lock can likely be maintained for a longer time. We also note that, with higher microcavity Q, the microcomb threshold power can be lowered and microcombs, with the similar FSR as our demonstration, with tens of milliwatt pump power has been implemented entirely on chip [SR3, SR4].

In addition to power modulation via the EOM, we also have a secondary feedback signal (derived by integrating the primary feedback control signal) to the EDFA which directly modulates the power, relatively slowly, primarily with the objective to increase the dynamic range of the lock (EDFA is not used as the sole feedback because it cannot be operated at the full feedback bandwidth). The feedback is designed in the above manner, with fast feedback via the EOM for high feedback bandwidth and slow feedback via the EDFA for high dynamic range, to preserve an optimal lock for a long period of time. Supplementary Fig. S5 summarizes the quality of the  $f_{rep}$  stabilization. After the stabilization of the comb spacing, we notice that the offset beat  $\xi$  also becomes more stable as can be visually observed from Supplementary Fig. S6. This is per our expectation of partial correlation between  $\xi$  and  $f_{rep}$  as described in the main text.



**Supplementary Figure S5. a,** RF spectrum of the stabilized beat note of  $f_{rep}$  with an RBW of 10 Hz. In the PI<sup>2</sup>D loop filter, the PI corner and differential frequency were both set at 200 kHz. The design provides a delicate compromise between noise suppression and loop stability. A remaining small noise oscillation at 205 kHz, however, is still present. **b**, Single-sideband phase noise of the reference 17.9 GHz local oscillator and the residual error from the  $f_{rep}$  phase-locked loop, showing

an excess phase noise of the stabilized comb spacing above 40 kHz from carrier. Inset: rms phase error integrated from 6 Hz to 600 kHz is 20 mrad.

The offset frequency  $\xi$  can be used as an indicator of pump frequency after stabilization of  $f_{rep}$ , as explained in detail in the main text. We therefore use this signal to stabilize the pump frequency when the comb spacing is locked. To achieve a high SNR (which is required to lock  $\xi$  effectively), we use an optical grating filter to select a 1-nm section of the comb where the beat frequency is strongest and then send that section to a photodetector to detect the beat (SNR is higher because of a strong beat note in the localized region and also because the detector is not saturated by the  $f_{\rm rep}$  beat note, which is much stronger when a larger region of the comb is considered). We send the output to a divide-by-15 frequency divider, and then downmix the signal with a local oscillator operating at 33 MHz to obtain the error signal. The offset beat  $\xi$  has more high frequency noise than  $f_{rep}$ , as we might expect, because it is affected not only by the pump frequency instability but also by high frequency noise in pump power that is not fully compensated by  $f_{rep}$  stabilization, the frequency divider is therefore necessary to reduce the high frequency noise and increase the efficacy of the lock. The error signal is sent to a PI<sup>2</sup>D lock box which provides a feedback signal to modulate the diode current of our ECDL which stabilizes the pump frequency. Similar to the feedback to lock  $f_{rep}$ , we use a slower secondary feedback (derived from the integrated primary feedback control signal) via the piezo controller of the ECDL to increase dynamic range and preserve the lock for a longer time. We have included a detailed discussion of the feedback locking mechanism in the Methods section of the main text.



**Supplementary Figure S6. a,** The measured offset beat  $\xi$  at an RBW of 100 kHz when the *f*<sub>rep</sub> is not stabilized. The high noise in the beat arises because the offset frequency  $\Delta$  depends on pump

frequency and intracavity power given by  $\Delta = \frac{1}{\sqrt{\pi c |\beta_2|}} \sqrt{\eta \left(n_g f_p - N \frac{n_g^2}{n_0} f_{rep} - \frac{\gamma c P_{int}}{\pi}\right)}$  and since  $\xi = \Delta - \left\lfloor \frac{\Delta}{f_{rep}} \right\rfloor f_{rep}$ , fluctuations in both pump frequency and  $f_{rep}$  add to instability in  $\xi$ . **b**, The measured offset beat  $\xi$  at an RBW of 10 kHz after  $f_{rep}$  is stabilized. We observe an increase in stability of  $\xi$  after stabilization of  $f_{rep}$  (and thereby stabilization of pump power). Residual noise in the beat note is due to pump frequency noise (and residual noise in pump power after  $f_{rep}$  stabilization).  $\xi$  can therefore be used to sense pump frequency fluctuations and stabilize it via feedback.

### Supplementary Note IV: Derivation of the modulation instability (MI) gain peak

We investigate the intracavity MI gain and derive the frequency at which it is maximum. Supplementary Equation S1, written below describes the cavity boundary conditions and Supplementary Equation S2 describes the wave propagation in the cavity when subject to chromatic dispersion and the Kerr nonlinearity [SR3]:

$$E^{n+1}(0,t) = \sqrt{\rho}E^{n}(L,t)exp(i\varphi_{0}) + \sqrt{T}E_{i}, \qquad (S1)$$

$$\frac{\partial E^{n}(z,t)}{\partial z} = -i\frac{\beta_{2}}{2}\frac{\partial^{2}E^{n}(z,t)}{\partial t^{2}} + i\gamma|E^{n}(z,t)|^{2}E^{n}(z,t), \qquad (S2)$$

Under the normalization  $U^n = \sqrt{\gamma L} E^n$ ;  $\kappa = \frac{z}{L}$ ;  $\tau = \frac{t}{\sqrt{|\beta_2|L}}$ , the NLSE is reduced to  $U_{\kappa}^n = -i\left(\frac{\eta}{2}\right)U_{\tau\tau}^n + i|U^n|^2U^n$  where  $\eta = \frac{\beta_2}{|\beta_2|}$ . (To model this [SR5-SR7], here we used the more general NLSE model instead the LLE, the latter which resides under the good cavity limit and the approximation that the evolution is slow compared to round trip time.) Assume steady-state continuous wave in the cavity, one such solution is:  $U^n(\kappa, \tau) = U_0 exp(i|U_0|^2\kappa)$ . A periodic fluctuation generated by instability is modelled by:

$$U^{n}(\kappa,\tau) = [U_{0} + v^{n}(\kappa) \times exp(i\Omega\tau) + v^{-n}(\kappa) \times exp(-i\Omega\tau)]exp(i|U_{0}|^{2}\kappa)$$

Here  $\Omega$  corresponds to the location of peak MI as will be derived subsequently. An important point to note here is that  $\Omega$  may not be an exact multiple of the (normalized)  $f_{rep}$ . Substituting this in the NLSE yields after some algebra that:

$$\begin{aligned} \frac{\partial v^{n}}{\partial \xi} exp(i\Omega\tau) &+ \frac{\partial v^{-n}}{\partial \xi} exp(-i\Omega\tau) \\ &= \frac{i\eta\Omega^{2}}{2} \left( v^{n} exp(i\Omega\tau) + v^{-n} exp(-i\Omega\tau) \right) \\ &+ i \left( \left( |U_{0}|^{2} v^{n} + U_{0}^{2} v^{-n^{*}} \right) exp(i\Omega\tau) + \left( |U_{0}|^{2} v^{-n} + U_{0}^{2} v^{n^{*}} \right) exp(-i\Omega\tau) \right) \end{aligned}$$

This then can be written as:

$$\frac{\partial}{\partial\xi} \binom{\nu^{n}}{\nu^{-n^{*}}} = \begin{bmatrix} \frac{i\eta\Omega^{2}}{2} + i|U_{0}|^{2} & iU_{0}^{2} \\ \\ -i(U_{0}^{*})^{2} & \frac{-i\eta\Omega^{2}}{2} - i|U_{0}|^{2} \end{bmatrix} \binom{\nu^{n}}{\nu^{-n^{*}}}$$

The general solution to the equation above can be written as:

$$\binom{v^{n}}{v^{-n^{*}}} = \binom{a^{n}}{b^{n}} exp(\mu\kappa) + \binom{c^{n}}{d^{n}} exp(-\mu\kappa)$$

With eigenvalue  $\mu = \Omega \sqrt{-\eta |U_0|^2 - \Omega^2/4}$  (S3) and eigenvector components satisfying:

$$\frac{a^{n}}{b^{n}} = \frac{-(U_{0})^{2}}{\left(\frac{\eta\Omega^{2}}{2} + |U_{0}|^{2} + i\mu\right)}; \frac{d^{n}}{c^{n}} = \frac{-(U_{0}^{*})^{2}}{\left(\frac{\eta\Omega^{2}}{2} + |U_{0}|^{2} + i\mu\right)}$$

Now including the cavity boundary conditions (by substituting  $E_n$  in Supplementary Eq S1) and noting that  $E_i$  is a constant, we see that we can write:

$$v^{\pm(n+1)}(\kappa = 0) = \sqrt{\rho} exp(i\varphi_0 + i|U_0|^2)v^{\pm n}(\kappa = 1)$$

Since

$$\binom{\nu^{n}}{\nu^{-n^{*}}} = \left[ \frac{\begin{pmatrix} 1 & 1 \\ \frac{\eta\Omega^{2}}{2} + |U_{0}|^{2} + i\mu \end{pmatrix}}{-(U_{0})^{2}} & \frac{-(U_{0}^{*})^{2}}{\left(\frac{\eta\Omega^{2}}{2} + |U_{0}|^{2} + i\mu\right)} \right] \binom{a^{n}}{c^{n}}$$

And

,

$$\begin{pmatrix} v^{n+1} \\ v^{-(n+1)^*} \end{pmatrix} = \sqrt{\rho} \begin{bmatrix} exp(i\varphi_0 + i|U_0|^2) & 0 \\ 0 & exp(-i\varphi_0 - i|U_0|^2) \end{bmatrix} \begin{pmatrix} v^n \\ v^{-n^*} \end{pmatrix}$$

We can write after substituting in:

$$\begin{pmatrix} a^{n+1} \\ c^{n+1} \end{pmatrix}$$
  
=  $\frac{\sqrt{\rho}}{t^2 - |s|^2} \begin{bmatrix} -|s|^2 & st \\ t^2 & -st \end{bmatrix} \begin{bmatrix} exp(i\varphi_0 + i|U_0|^2) & 0 \\ 0 & exp(-i\varphi_0 - i|U_0|^2) \end{bmatrix} \begin{bmatrix} e^{\mu} & e^{-\mu} \\ te^{\mu}/s & s^*/(at) \end{bmatrix} \begin{pmatrix} a^n \\ c^n \end{pmatrix}$   
where  $s = -(U_0)^2$ ;  $t = \frac{\eta\Omega^2}{2} + |U_0|^2 + i\mu$ ;  $\vartheta = \varphi_0 + |U_0|^2$ .

Taking the determinant of this matrix we arrive at the eigenvalues as:

$$q_{\pm} = \sqrt{\rho} \left( p \pm \sqrt{p^2 - 1} \right) \quad \text{where } p = \frac{-|s|^2 (e^{-\mu} e^{-i\vartheta} + e^{\mu} e^{i\vartheta}) + t^2 (e^{\mu} e^{-i\vartheta} + e^{-\mu} e^{i\vartheta})}{2(t^2 - |s|^2)}$$

Under the mean field approximation, which in this case means that  $\mu \sim O(\varepsilon)$ , we can approximate  $e^{\mu}$  to first order as 1+ $\mu$ , rewriting this equation  $p = \cos(\vartheta) - i\mu\sin(\vartheta)\frac{(t^2+|s|^2)}{(t^2-|s|^2)}$ . Now we see that

$$t + |s| = \frac{-\mu^2}{2\eta\Omega^2} + i\mu \quad \text{and} \quad \text{utilizing} \quad (S3), \quad \text{we} \quad \text{can} \quad \text{rewrite} \quad i\mu\mu\sin(\vartheta)\frac{(t^2 + |s|^2)}{(t^2 - |s|^2)} \quad \text{as}$$
$$i\mu\sin(\vartheta)\frac{(\frac{-\mu^2}{2\eta\Omega^2} + i\mu)}{(\frac{-\mu^2}{2\eta\Omega^2} + i\mu - 2|s|)} - i\mu\sin(\vartheta)\frac{2t|s|}{(\frac{-\mu^2}{2\eta\Omega^2} + i\mu)(\frac{-\mu^2}{2\eta\Omega^2} + i\mu - 2|s|)} \quad (S4)$$

Now the first term of (S4) is of order  $\mu^2$  and is neglected while the second term reduces to  $t = \frac{\eta \Omega^2}{2} + |U_0|^2 + i\mu$  and since  $\mu$  is small, this is simply  $\frac{\eta \Omega^2}{2} + |U_0|^2$ . So  $p = \cos(\vartheta) - (|U_0|^2 + \frac{\eta \Omega^2}{2})\sin(\vartheta)$  (S5). We note that  $q_+ = \sqrt{\rho} \left(p + \sqrt{p^2 - 1}\right)$  is the (largest) eigenvalue of interest and takes its maximum for large p.

Now we write  $\vartheta = 2m\pi - \delta + |U_0|^2$  where  $\delta$  is the detuning from resonance. Applying the good cavity limit we state that both the detuning  $\delta$  and the additional phase added per round trip due to self-phase modulation  $|U_0|^2$ , are small in comparison to  $2\pi$ . Further the good cavity limit ensures that  $\rho \rightarrow 1$ , we introduce the parameter  $\theta = 1$ - $\rho$  where  $\theta \sim O(\varepsilon)$ . Subsequently (following [SR5]) we find that to first order the eigenvalue  $q_+$  can be written as

$$q_{+} = 1 - \frac{\theta}{2} + \sqrt{4\left(\delta - \frac{\eta\Omega^{2}}{2}\right)|U_{0}|^{2} - \left(\delta - \frac{\eta\Omega^{2}}{2}\right)^{2} - 3|U_{0}|^{4}} \quad (S6)$$

And the maxima of this eigenvalue is obtained at  $\Omega_{opt} = \sqrt{2 \eta (\delta - 2|U_0|^2)}$  which is the frequency at which MI gain is maximum.

Now  $\delta$  can be written as  $\frac{2\pi(f_p - f_0)}{f_{rep}}$  where  $f_p$  is the pump frequency,  $f_{rep}$  is the comb spacing and  $f_0$  is the resonance frequency,  $f_{rep}$  can also be expressed as  $c/(n_g L)$  and  $f_0 = N \frac{n_g}{n_o} f_{rep}$  where  $n_g$  is the group index and  $n_o$  is the refractive index. Furthermore  $\omega = \Omega/\sqrt{|\beta_2|L}$  where  $\omega$  is the frequency with respect to real time coordinates and  $|U_0|^2 = \gamma L |E_0|^2$  where  $|E_0|^2$  is the intracavity power, denoted by  $P_{int}$ .

Putting these together we have:

$$\omega_{\text{opt}} = \sqrt{\frac{4\pi n_{\text{g}} \left(f_{\text{p}} - N \frac{n_{\text{g}}}{n_0} f_{\text{rep}}\right)}{\eta |\beta_2| c}} - \frac{4\gamma P_{\text{int}}}{\eta |\beta_2|} \quad (S7)$$

This formula is intended as an approximation for the case when there is no mode interaction and close to the onset of MI. It illustrates that the breather frequency  $\xi$  is dependent only on the two

parameters  $P_{int}$  and  $f_p$  controlled by pump power and pump frequency respectively. Therefore when the pump power is locked via feedback (utilizing  $f_{rep}$  as the indicator of intracavity power),  $\xi$  is only dependent on pump frequency  $f_p$ . The addition of periodic mode interaction and the effects of "mode pinning" after breather comb formation introduce further complications that can be simulated in the LLE.

To perform the simulations, we first introduce a periodic mode interaction in the LLE via mode shifts (following [SR8]) at resonances every 4 nm. The generated comb spectrum is plotted in Supplementary Fig. S7a, we note the presence of equally spaced subcombs that are on the verge of merging similar to our generated spectrum. Supplementary Fig S7b, plots the comb evolution with detuning swept over 100 MHz, illustrating the large stability region of our comb. We then



**Supplementary Figure S7. a,** The simulated breather comb spectrum with multiple equally spaced subcombs. **b,** Evolution of the comb spectrum in the cavity with detuning swept over 200 MHz. The spectrogram shows onset of MI at mode 20, which is disrupted in the simulation due to mode interaction and hence allows the comb to be seeded when dispersion is normal at 30 fs<sup>2</sup>/mm. **c**, Shows the breather frequency  $\xi$ , when pump detuning is increased and intracavity power is kept constant via a simulated PID loop. We note here that since  $\xi$ , depends *both* on the pump detuning and pump power, locking the power provides a direct relationship between the pump and primary comb line, which manifests as the breather is plotted in time.

note that the breathing frequency can in fact be seen as a slow relative phase oscillation between the pump and the first primary comb line as plotted in Supplementary Fig. S7d. A single cycle of this phase oscillation corresponds to a frequency of about 354 MHz, the breather frequency. In parallel to the phase, there is also a slow intensity oscillation of the comb line at the same 354 MHz that corresponds to energy being transferred from the primary lines to the pump and back. Now, finally to simulate the relationship between breather frequency and pump frequency, in conditions similar to our experiment, we simulate a PI loop to control the intracavity power P<sub>int</sub> using only feedback to pump power. We note here (and in the main text) that since the frep is only dependent on cavity temperature [SR9], which in turn is entirely dependent on P<sub>int</sub> (neglecting environment temperature changes), locking  $f_{rep}$  via feedback to power must necessarily lock  $P_{int}$ . This simulated loop therefore exactly corresponds to the experimentally demonstrated  $f_{rep}$ stabilization loop via pump power feedback. The PI corner and bandwidth of the simulated feedback loop is however set at 18 MHz (rather than 200 KHz), for ease of simulation to prevent unreasonably large number of round trips required for the loop to stabilize  $P_{int}$ . We subsequently ran the simulation for 5 million roundtrips with PI loop to stabilize P<sub>int</sub> engaged and swept laser detuning while monitor change in breather frequency. Our results are plotted in Supplementary Fig. S7c, the simulated slope of change in breather frequency to change in pump detuning is  $\sim 45$ KHz/MHz.



# Supplementary Note V: Out-of-loop characterization

**Supplementary Figure S8. a,** To quantify the frequency instability of the Kerr frequency comb, two comb lines (pump at 1598 nm and *i*<sup>th</sup> comb at 1555 nm) are compared to an independently stabilized FFC and the heterodyne beat frequencies are counted with a 10-digit,  $\Lambda$ -type frequency counter. The FFC is referenced to a rubidium-disciplined crystal oscillator with a frequency fractional instability of  $5 \times 10^{-12}$  at 1 second. The gratings critically remove the unwanted reference FFC comb lines for reliable counting measurements. **b**, The repetition rate of the FFC ( $\approx 250$  MHz) is detected with a PD and locked to an RF local oscillator, in addition, *f-2f* interferometry [SR10-SR12] is used to detect *f<sub>ceo</sub>* and lock it to an RF reference with the same clock as that used to lock *f<sub>rep</sub>*. **c**, The Allan Deviation of the *f<sub>ceo</sub>* is plotted for the FFC in mHz and the Allan Deviation for *f<sub>rep</sub>* is plotted relative to the carrier. As we observe from the plot, *f<sub>rep</sub>* is the limiting factor for the stability of our reference, which is per our expectation, because of the high sensitivity of the comb line frequencies to *f<sub>rep</sub>* due to the low optical division ratio ( $\approx 10^{-6}$ ).

## Supplementary Note VI: Verification of $\xi$ across different chips and breather states

Formation of Kerr combs with a single offset beat in addition to  $f_{rep}$  is not a unique property dependent on microresonator characteristics but in fact is general and arises from the mechanics of Kerr comb generation. These combs have also been observed previously in microresonators [SR1, SR13], but with very different characteristics. In addition, to the 18 GHz comb described in the main text, we also observe a similar state in a single mode Si<sub>3</sub>N<sub>4</sub> microresonator cavity with a tapered structure [SR14] thereby verifying the applicability of our approach across different breather comb states and chipsets. The comb spectrum is shown in Supplementary Fig. S9a. and the offset beat is measured in different comb slices, as shown in Supplementary Fig. S9b, to verify that it is truly a single offset comb state. Breather combs in general have a modulated spectrum such as in Supplementary Fig. S9a, this may limit performance in some applications such as the spectral density in dense-wavelength optical communications. However, breather solitons can provide both the breathing frequency for locking while having a smooth spectrum for various applications for further studies.



**Supplementary Figure S9. a,** Spectrum of a comb-state, similar to the one we stabilize, generated in a single mode microresonator with a tapered structure. This comb state generates a single offset beat  $\xi$  in the RF domain in addition to the repetition rate. **b**, To verify that the offset frequency is uniquely defined across the whole Kerr frequency comb, we measure it at various different spectral segments with a tunable filter (0.22 nm FWHM filter bandwidth). Free-running  $\xi$  without  $f_{\text{rep}}$  stabilization ( $\approx$  700 MHz) in different spectral regions is measured to be the same within error bars of  $\approx$  200 kHz. At wavelengths where the beat notes have SNR higher than 10 dB (100 kHz RBW), 10 measurements are taken to determine the mean value of the offset frequency. The error bar of the measurement is defined as the peak-to-peak deviation from the 10 measurements.

Supplementary Note VII: Verification of stabilization of  $f_{rep}$  stabilization after locking  $\xi$ 



Supplementary Figure S10. a,  $f_{rep}$  measured at an RBW of 1 kHz before locking  $\xi$  but after locking pump frequency, b,  $f_{rep}$  measured after locking  $\xi$  at an RBW of 1 kHz. Locking sidebands are observed at  $\approx 200$  kHz, corresponding to the PI corner of the feedback loop intended to stabilize  $\xi$ . Stabilization of  $f_{rep}$  is clearly observed subsequent to locking  $\xi$ . Furthermore, as explained in the

main text, we expect the phase noise of this signal to be  $\approx 38$  dB lower than the phase noise of the locked signal  $\xi$  after division to the same carrier.

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