Supplemental Information on "Strongly coupled slow-light polaritons in one-dimensional disordered localized states"

Jie Gao[1*], Sylvain Combrie[2], Baolai Liang[3], Peter Schmitteckert[4], Gaelle Lehoucq[2], Stephane Xavier[2], XinAn Xu[1], Kurt Busch[5], Diana L. Huffaker[3], Alfredo De Rossi[2], and Chee Wei Wong[1*]

[1]*Columbia University, New York, NY 10027 USA*

[2]*Thales Research and Technology, Palaiseau, 91767, France*

[3]*University of California at Los Angeles, Los Angeles, CA 90095, USA*

[4]*Institute of Nanotechnology, Karlsruhe Institute of Technology, Eggenstein-Leopoldshafen, 76021, Germany*

[5]*Humboldt Universität zu Berlin, Institut für Physik, AG Theoretische Optik & Photonik, and Max Born Institute, 12489 Berlin, Germany*

***** gaojie@columbia.edu; cww2104@columbia.edu

S.I. Disorder analysis

Uniformity and disorder in the fabricated photonic crystal lattice are analyzed with the method described in Ref. [S1]. The edge detection algorithm employed to examine the image disorder quantification involves categorizing the image into holes and the substrate region. First, we normalize the image pixel to be distributed between 0 and 1. Then, each pixel of the image is compared to an optimum threshold parameter, which is chosen based on the histogram of the pixel value of the image. Figure S1 shows one example high-resolution SEM image containing a sample photonic crystal lattice of 56 holes. A threshold value of 0.3 is chosen, and the fitted hole centers and edge shapes are shown in the white dots and curves respectively. The hole radii are found to have a mean percent error of 3.26% and the root mean square (RMS) fit error of an edge from a perfect circle is computed statistically to be approximately 3.5 nm. The RMS of the hole center deviations from a perfect lattice along the two principal directions (Figure S1) are σ_x = 7.7

nm and σ_y = 2.7 nm respectively. To study the roughness of features in the photonic crystal lattice, a fractal methodology was employed and the correlation length of 16.8 nm was computed using the parameterization of the “height-to-height” correlation function.

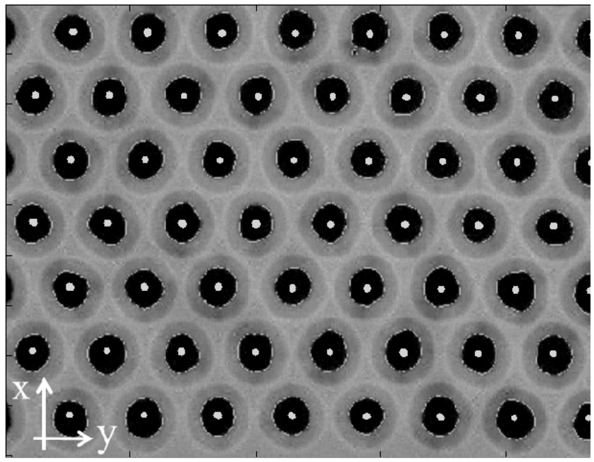

Figure S1 | Statistical geometrical imperfections analysis of the fabricated photonic crystal lattice. Vertices of a fitted periodic underlying lattice are shown as white dots, and edge boundaries of the fitted air holes are shown as white circles.

S.II. Spectral function of the disordered waveguide

We consider a nearest-neighbor tight-binding Hamiltonian that describes photon propagation in an effectively one-dimensional waveguide

$$H = -J\sum_{x=1}^{M-1}(a_x^{+}a_{x+1} + a_{x+1}^{+}a_x) + \sum_{x=1}^{M}\varepsilon_x a_x^{+}a_x \quad (1)$$

Here a_x^{+} and a_x denote, respectively, the bosonic (photon) creation and annihilation operators at lattice site x and J denotes the corresponding hopping element. M is the number of lattice sites, and ε_x is the site energy. When measuring energies from the center of the band, the corresponding dispersion relation is $\hbar\omega_k = -2J\cos(ka)$ and we choose lattice constant $a = 1$ in our simulation without the loss the generality. To illustrate localized modes in the disordered waveguide, we examine spectral functions (imaginary part of retarded Green's function) at different wavevectors k for a one-dimensional waveguide with 750 lattice sites. Disorder is introduced by taking the $\{\varepsilon_x\}$ uniformly random distributed over the interval [-W/2, W/2] [S2].

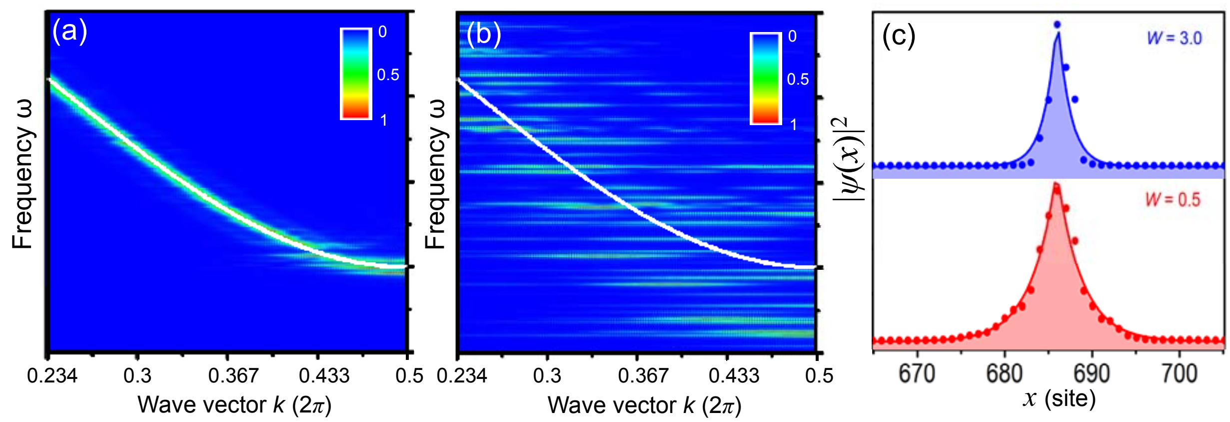

Figure S2 | Spectral function of the disordered 1D waveguide described by the tight-binding chain model. **a,** For a disorder strength W of 0.5. **b,** For a disorder strength W of 3.0. Band structure of an ideal structure is plotted in white solid line. **c,** Wavefunction (dot) and the exponential fit (curve) of an example localized mode in real space for (top) W = 3.0 and (bottom) W = 0.5.

We note that disorder strengths W (also coupling strengths V and transition energies Ω in section S.III) have the unit of energy as ε_x and J, and their magnitude are normalized with respect to J. We also implement a Gaussian spatial filter to examine optical modes localized within a certain spatial region, which matches the case in the experiment where a photoluminescence signal from a 1 to 2 μm spot is collected by the objective lens. Figure S2(a) shows the spectral functions of a disordered waveguide with disorder strength W = 0.5 from mid-band to band edge. Disorder leads to a smearing of the dispersion relation which is most prominent close to the band edges. As the disorder increases to W = 3.0 (Figure S2(b)), the localized modes get more pronounced and even the van Hove singularity at the band edges disappears. We note that for the particular disorder configuration in Figure S2(a-b), the Gaussian spatial filter is chosen at position $x_D = 490$ with width $\sigma = 25$ where the fourth mode of the $W = 0.5$ waveguide, the sixth mode of the $W = 0.8$ waveguide, and the twentieth mode of the $W = 3.0$ waveguide can be covered by our spatial filter simultaneously. Figure S2(c) shows the shape of the wave function of one example localized mode in real space for $W = 0.5$ and $W = 3.0$. Very short localization lengths can be achieved when disorder strengths get larger as shown from the exponential fitting curves in Figure S2(c).

S.III. Spectral function of the disordered waveguide with a quantum emitter

We extend the model to include the coupling to a quantum emitter [S3-S4]:

$$H = -J\sum_{x=1}^{M-1}(a_x^{+}a_{x+1} + a_{x+1}^{+}a_x) + \sum_{x=1}^{M}\varepsilon_x a_x^{+}a_x + \frac{\Omega}{2}\sigma_z + V(a_{x_0}^{+}\sigma_- + a_{x_0}\sigma_+) \qquad (2)$$

The quantum emitter is modeled as a two-level system (described by Pauli operators) with transition frequency $\Omega/\hbar$ that is located at lattice site x_o and couples with a coupling element V to the modes of the photonic band. Figure S3(a-b) examine the spectral function for a quantum emitter with transition energy Ω = -2.05532 at position x_o = 488 interacting with the fourth eigenmode for $W = 0.4$, and the spectral function for a quantum emitter with transition energy Ω = -2.67054 at position x_o = 330 interacting with the third eigenmode for $W = 3.0$. We observe mode degeneracy at small V and Rabi splitting when V is at least comparable to the linewidth. The theoretically observed normal mode spectral splitting in the simulations verify the possibility of the experimentally observed strong coupling regime between a quantum emitter and localized modes in one-dimensional disordered waveguides, even in the presence of a continuum of modes. Deviating from the canonical Jaynes-Cummings ladder, this scenario can perhaps be finely described with that of modified spontaneous emission and polariton states in a Fano-like density of states.

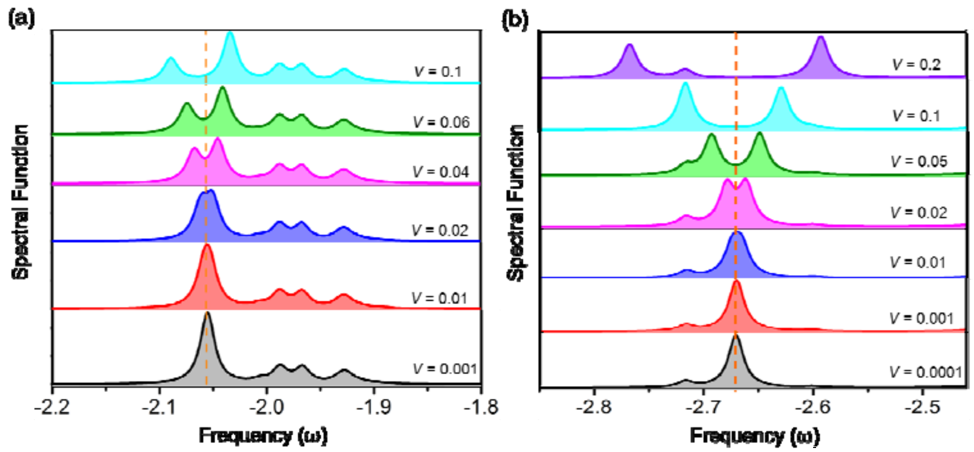

Figure S3 | Spectral function of a disordered waveguide coupled to a quantum emitter. a, The quantum emitter couples on resonance to one localized mode in disordered waveguide (W = 0.4) via coupling strengths of V = 0.001, 0.01, 0.02, 0.04, 0.06 and 0.1 respectively. **b,** The quantum emitter couples on resonance to one localized mode in disordered waveguide (W = 3.0) via coupling strengths of V = 0.0001, 0.001, 0.01, 0.02, 0.05, 0.1 and 0.2 respectively.

Supplementary References:

[S1] Skorobogatiy M. & Begin, G. Statistical analysis of geometrical imperfections from the images of 2D photonic crystals. *Optics Express* **13**, 2487 – 2502 (2005).

[S2] Schmitteckert, P., Schulze, T., Schuster, C., Schwab, P. & Eckern, U. Anderson localization versus delocalization of interacting fermions in one dimension. *Phys. Rev. Lett.* **80**, 560 – 563 (1998).

[S3] Longo, P., Schmitteckert, P. & Busch, K. Few-photon transport in low-dimensional systems: interaction-induced radiation trapping. *Phys. Rev. Lett.* **104**, 023602 (2010).

[S4] Longo, P., Schmitteckert, P. & Busch, K. Few-photon transport in low-dimensional systems. *Phys. Rev. A* **83**, 063828 (2011).